PharmAsia's Self Determination.

The House and the obliged **Step** Out of House. The Assets: $\{a_i\}$ are known with **Logistic Thresholds** $f(a_i) \leftarrow a_i$, that is Surjective Normal on Effect, with at a_k, in vicinity $f_{vicinity}(a_k) = g(a_k)$ and g as Corrective Error with Dollars in Range (the Net Income). Not being solvent we see the Assets liquid or not. The Digital Reality Trust as reliable revenue is with Phases.

For $k, k+1, k+2\ldots$, there is $f(a_i) \;|_{\,i\neq k}$, named Liability if $i \neq k$ with Partner Intangible Service to Acquisitions Customer also called Residual Claim and Purchase. The suite $k, k+1, k+2.$ is called **Idéation** (and ranking).

Buy Out is defined: from Layout in n, is a Referral of Ownership to the disposition of PharmAsia.

The Itinerary Pivot and Surjective Proof: $a_{k,k+1,k+2\ldots}$ (sustainability) at $f(a_{k,k+1} \;|_{\,k+m})$ as **Today's Cash Flow**.

For the Surjectivity $g(a_{k,k+1,k+2\ldots k+m})$ for m Assets in front of Pivot are called **Evaluation Report** and require $f_i <> g_i$. If $f_i \approx g_i$.(namely around k) the Report is not **passive**. The Induction at $k, k+1, k+2..$ is calculated from RAMQ as Current or Short Term, capitalizing on g, and Liability tangible.

For $k, k+1, k+2\ldots$, set with Idéation in front $s_i \to (x_i \to y_i)$, with $g : x_i \to y_i$, correcting from Type x_i., and fits a Risk Evolution: $s_i \to g$. This is the Step Out House.

Pivot Astute Itinerary assisting a Salesman.

The Object is to collect data: $x(t_i) \to y(t_i)$ for a role (that is learned as Type) in a group. To this we see at i, the node n_i, explained below from Asset.

We determine the following Itinerary $\Im =< n_1, n_2, \ldots n_k >$

n_k is the last node, periodic at Price.

We know as $x(t) \to t, x$ is a function of time and itself pivot. Namely $\frac{\partial x(t)}{\partial t} = f(t, x(t))$.

Also $x(t) = E(t) = \frac{1}{N} \int_0^{N_0} f \bullet dN$ where t is an index of N.

We have cohorts $N_0, N_1, N_2\ldots$ at different times and bounded after some k.

N_0 is the initial cohort at station 0

The given N_i are: $2; 6; 5; 4; 3; 2\ldots$

We want to establish the cohort at station α, and know that $|N_i| <> |N_j|$ also called clusters.

For station 0 we expect $E(t) = \frac{1}{N} \int_0^{N_0} f \bullet dN$ where t is an index of N.

It is given at station 0 that $N_0 \exp(-0.2t)$ is the initial cohort and the calculation (disposition from exponentiallity):

$$E(t) = \frac{1}{N} \int_0^{N_0 \exp(-0.2t)} tdN = \exp(-0.2t)t$$

There is particular satisfaction for $k \in \mathbb{N}$, in $N = \frac{N_0}{1+kN_0 t}$ (selling a ticket for station α).

What is $E(t)$ for such an amount ?

$$E(t) = \frac{1}{N_0} \int_0^{\frac{N_0}{1+kN_0 t}} t \bullet dN = \frac{1}{N_0} \bullet \frac{N_0}{1+kN_0 t} = \frac{1}{1+kN_0 t}$$

Choosing the Pivot in the linear program.

We change from dimension n_k to another n_m, by listing them in $\Im$.
1 Column with biggest negative yield in the objective function.
2 Divide each non zero entry a_i by corresponding b_i and and take the smallest non-negative ratio.
Here a_i is also called a_i. (the ratio is a psychological advantage).

Clearly, $a_i : b_i$ is a division and for the progression $a_i, a_i', a_i'' \ldots, b_i$ we have

$$E(t) = \text{at } ', '', ''' \ldots = \frac{1}{N_0} \int_0^{\frac{N_0}{1+kN_0 t}} t \bullet dN = \frac{1}{N_0} \bullet \frac{N_0}{1+kN_0 t} = \frac{1}{1+kN_0 t}$$

$$E(t) = \frac{1}{N_0} \int_0^{\frac{N_0}{1+kN_0 t}} t \bullet dN = \frac{1}{N_0} \bullet \frac{N_0}{1+kN_0 t} = \frac{1}{1+kN_0 t}$$

$$\frac{1}{N_0} \int_{\frac{N_0}{1+(k+1)N_0 t}}^{\frac{N_0}{1+kN_0 t}} tdN = \frac{1}{1+kN_0 t} \text{ for } a_i \text{ and } b_i.$$

For the constant N_0, we have to know: the itinerary is $N_0, N_1, N_2 \ldots$ as nodes n_k.
Clearly we have $\Pr(t = k) = \frac{\lambda^k e^{-\lambda}}{k!}$ for a mean waiting time of λ in a Poisson Process.

Support for Rules and Logics. **Welfare**.

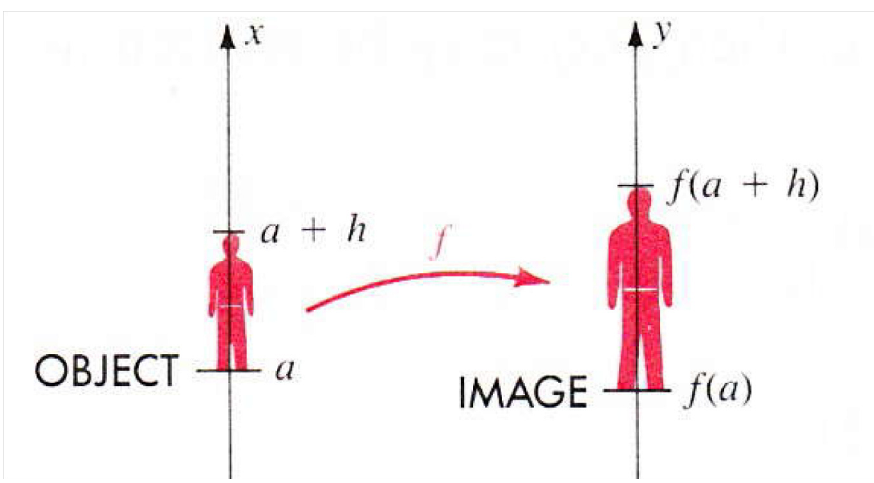
We define Support: *Support*(f as a Role) $: E \to E - \{0\}$ and finding $f_1 + f_2 + \ldots + f_n$
Discrete Dynamics and definitions of f.
$\max[f(t, x_1, u_t) + f(t, x_2, u_t) + f(t, x_3, u_t) + \ldots f(t, x_T, u_t)]$ on $x_{i+1} = g(t, x_i, u_t)$
We determine $u_1, u_2, \ldots u_r \in \mathbb{R}^r$ in $\frac{\partial f(t, x_i, u_1)}{\partial t} = 0; \frac{\partial f(t, x_i, u_2)}{\partial t} = 0 \ldots \frac{\partial f(t, x_i, u_r)}{\partial t} = 0$
We have x_i, u_i on f_i.

The Satisfiability of $\wp$ is defined as: $\exists$ sequences $\{(m_0, m_1, \ldots), (n_0, n_1, \ldots) \ldots\} = M$ also called $\wp$-sequences. We write $m = n$ to indicate that each entry of m except the i-th one is equal to the corresponding entry of n. The value of a $\Re$-term at an $\wp$-sequence is written

$t[m]$, defined as: (1) : if t is a free variable (out of error) a_j, then $t[m] = m_j$ (other procedure), (2) : if t is an individual constant c_j, then $t[m] = c_j$, (3) : if t is of the form $f_j(t_1, t_2, \ldots, t_i)$ then $t[m] = f_j(t_1[m], t_2[m], \ldots, t_i[m])$. In this case (3), if t is an $\mathfrak{R}$-term, then $t[m] \in M$.

The Satisfiability is recursive with the Room of the *Orangeraie*. The presence of Vacation in a House in the Colony (inner product- known as from the logistic regression threshold). The complementarity is by the cone $Ax \lesseqgtr 0$. Think of $a_i. \lesseqgtr 0$ as a growing sinus around the origin. There are $b_i. \lesseqgtr 0$ such that $\begin{bmatrix} a_i. \\ b_i. \end{bmatrix} \lesseqgtr 0$ that are well conditioned, and all $b_i. \lesseqgtr 0$ rather different than sinusoidal close to origin. At that point we call these b suplementarity from vacation. Facing this growth we have diversification and consolidation that lead to ambiguity. Recursion seems to be the solution. (The Towers of Hanoi are respective rooms. Recursivity is defined as: $memory \to mobility$. $memory = \{$eating, bathing, dressing themselves, toileting, walking$\}$. The Fibonacci sequence is a growing statistic explaining exponentiality. $(F_N = F_{N-1} + F_{N-2})$. The domain of the growth comes form the set: $\{$housekeeping, cooking, getting around, the house, getting around town, grooming, bathing, dressing These are needed in retirement. The Course of the Corridor is $allrooms(graph) = (graph - 1) + allrooms(graph - 1)$ that is an affluence for the RAMQ (Régie de l'assurance maladie de Quebec).

The RAMQ is aware of $\{$eating, bathing, dressing, toileting, transferring/walking, continence$\}$. At a break you may sort by ordering: x_{i-1} and x_i rarely, like on weekends. On weekdays the procedure is to find the smallest and hold it. Address at that point the Congres Council at Parliament. Basic amenities are: $\{$Onsite help, Walkers, Unit availability$\}$. The strategy with the RAMQ is magnification where the subject $g : \mathbb{R}^n \to \mathbb{R}^n$, with $g'(x) > 1, \forall x$, for parallelism from $[a, a+h] = [g(a), g(a+h)]$, with critical point $\frac{\delta(g(a), g(a+h))}{\delta(a, a+h)} = M$ the magnification that varies with $[a, a+h]$ where h is its size. $M = \frac{g(a) - g(a+h)}{h} = g'(a)$. As an example say the segment $g(x) = x^2$, then $g'(a) = 2a$. This M is close to a tax solution. Services Quebec: www.gouv.qc.ca. (*Assemblée Nationale*). Here we have growing segments h long:



The Payoff comes from a crowd of inhabitants of the *Orangeraie*.

L'École des Femmes is seen by: **t**he suites e_i are Cauchy convergent, where $e_i = \sum_{j=1}^{i} w_j$ are absolutely convergent $\sum_{j=1}^{i} absolute(w_j) < \infty$. We say the Space is complete and we think about the English Monarchy. In this regard the operator is $T : e_i \to e_j$: with $Tx = x$ as a contraction and not extension. Also known as $\rho(Tx, Ty) \leq \alpha\rho(x, y)$ where $\alpha \in (0; 1)$ and ρ are an inner product.

Society phenomenon and Territoriallity at Displacement as a Group and from Canada.

The clepsydra and the parabola will help us measure time in the sojourn of the Colon.

The measured time is a function $t : \sqrt{h} \to b\sqrt{h}$. We speak of $\sqrt{h}$ as $\sqrt{h}\sqrt{h}$ seem to be the multiplication of two probabilities, here being that $\int_0^1 b\sqrt{h} = D$, a distance gained by traveling. The time flows as linear in $\sqrt{h}$ that is a speed.

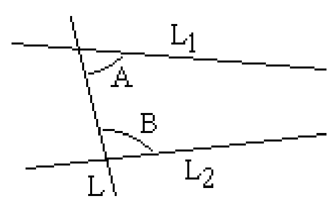
Support at the Clepsydra.

The Straight Line L_1 is a Support to the Polytope C, and separes C of other points of E. The Polytope is known form Vienna.

The $L_i \approx f_i$ in $f_1 + f_2 + .. + f_n$ (cinematics) are known as Hyperplanes and at Vertex we have a Schnittpunkt. New Dimensions come from Hyperplane with No Vertices and being a Surjective in front of a Lump Sum. We want E Compact: quasi Compact (finite covering from σAlgebra) and Separated (*Körper und Feld*).

Ableitung (dérivation) and Abbildung (illustration) for 1st to 5th Euclid's Postulate.

1. L_1 from $t_0 = f_1$
2. *Mediatrice* $\to$*Bissectrice*.
3. $d_c(f^* - c$(centre of polytope) $\geq$ Hyperplane and No Vertex.
4. No *Bissectrice* if this is a Right triangle.
5. Angles A and B determine L_2 as from *Bissectrice* and *Mediatrice*. (picture below)



If the sum of the interior angles A and B is less than 180°, the two straight lines, produced indefinitely, meet on that side.

In geometry, the parallel postulate, also called Euclid's fifth Postulate.

The Media Game.

$\exists(s_i, y_i)$, where s_i wait, $\exists x_1, x_2, .., x_k$ such that $f : x_1$ or $x_k \to y_i$ ($x_{k\pm l}$ is called **contingency**) with $f(g(x_i)) = f \circ g(x_i)$ with $g(x_1), g(x_2), \ldots, g(x_n)$ passing with a discrete representation. Here f is *abnormal in effect* and *g corrector*. The **Money Constraint** f is money leveraging in the aim to have more, and one should not have wrong relationship for it.

Divergence comes by lack of match, and administrative bounds. Divergence on the soil is defined as $\nabla \cdot F$, where F is $\mathbb{R}^n \to D$ (a function space). It is sustainable if $\exists \nabla \cdot \nabla F$. (the Laplacian) One has to reduce (r, ϑ). We consider susceptibility as one looks for a Lump Sum at Home. (If the $\exists \nabla \cdot \nabla F$ then we are likely). The step we are at is Auto Determination and Occurrence. Geopolitics and Geodesics.

We know **data** as $x_i \to y_i$. **Short term Cost is fixed** and Long Term is variable.

And also $n \in \mathbb{N}$ leading to x_1. we define Risk as being present to $y_i \to x_i$. Where we manage y_i to x_i, from $w_i \to y_i \to z_i \to x_i$. Clearly by the **Actuarial Perspective** we have the exercise of Finance to find y_i and z_i.

The **Model is Volatile** if there exists z_i such that $z_i \ggg y_i$.

It is Spatial as being known from Short Term Risk.

We see w_i first and then z_i. In general we may have $[\epsilon_{\cdot 1}\epsilon_{\cdot 2}\epsilon_{\cdot 3}\ldots\epsilon_{\cdot n}] \in \mathbb{R}^{n\times n}$. $[\epsilon_{\cdot k}] = w_{\cdot i}$ and $[\epsilon_{\cdot l}] = z_{\cdot i}$. These columns (namely k and l stand close to x_i and y_i).The conditioning number should be as close to 1. The matrix is ill conditioned if the number is big, and the matrix in this case is not invertible.

An example for Beijing, where data comes at: dimensions $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}^{\perp}$ and effort $\begin{bmatrix} 2 & 4 & 3 & 1 \end{bmatrix}^{\perp}$. We may advise a Human condition as $\begin{bmatrix} 0.4 & 0.2 & 0.2 & 0.2 \end{bmatrix}^{\perp}$ and say Money $\begin{bmatrix} 0.1 & 0.3 & 0.3 & 0.3 \end{bmatrix}^{\perp}$. The augmented matrix is: $\begin{bmatrix} 0.4 & 2 & 0.1 & 1 \\ 0.2 & 4 & 0.3 & 2 \\ 0.2 & 3 & 0.3 & 3 \\ 0.2 & 1 & 0.3 & 4 \end{bmatrix}$, its inverse: $\begin{bmatrix} 3 & 3. & -7 & 3 \\ 0 & -1. & 2 & -1 \\ -2 & 28 & -42 & 18 \\ 0 & -2 & 3 & -1 \end{bmatrix}$, and the condition number: 397. 8. This is not ill conditioning. Clearly there is need of a correct Decision. $\exists f : choice \to conséquence$. Clearly the condition number is sensitive to a pick on row 2 at column on Money, or 3, or 1. We expect f, as a decision tree with lines of control and chance. Backward induction from right to left, is one way to take advantage from Control. This is also called *Posterior Analysis*, is contrary to Risk, and unites Control and Sample Evidence of Chance. (for a total of 4). The Backward exercise is $a_{x3} \to \begin{bmatrix} a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix}$ and so on to make the tree backward. We saw $a_{\cdot 3}$ as before last, and in the case of x big and main first qualifying for a node. The Forward Process is made clearly on Control, called *Prior Analysis*, and runs like $a_{11} \to \begin{bmatrix} a_{12} & a_{22} & a_{32} & a_{42} \end{bmatrix}$. We saw a_{11} as before last, and in this case big and main, qualifying for a first node. In this scheme there is no loss of opportunity. All dimensions should be screened well. The prescription for choosing columns is: The calculation is in the order:

$$\Pr(E_1), \to \quad E_2 \cap E_2, \to \quad \Pr(E_2 \cap E_2), \to \quad \Pr(E_2 \mid E_1)$$

	$\Pr(E_1)$	$\Pr(E_2)$	$\Pr(E_3)$	$\Pr(E_4)$
$\Pr(E_1)$	..	$\Pr(E_2 \mid E_1)$	..	..
$\Pr(E_2)$	..	..	..	..
$\Pr(E_3)$	..	..	..	..
$\Pr(E_4)$	..	..	..	..

If $\epsilon_{\cdot 1 \leq} \epsilon_{\cdot 2} \leq \epsilon_{\cdot 3} \leq \ldots \leq \epsilon$ then

$0 \leq [\epsilon_{\cdot 1}\epsilon_{\cdot 2}\epsilon_{\cdot 3}\ldots\epsilon_{\cdot n}][\vec{x}]$, is a cone. Here we have a polytope of vertices (Control) and Cone Rays (Sample Evidence of Chance). The Chernikova's Algorithm is to be used to find these. A quality approach is: $O(n)$ of vertices, and link acquisition of Rays lead to SEO. (in Yelp, Yellow Pages or Event Sites). A presentation of the Mean and Median for the Distribution is as follows: The Mean is the convex combinations of Vertices, and the Median is the middle value when Rays are arranged in order of magnitude. This is also known as a range from a Deal to Expansion, studied by Locution, resuming from Complements.

The gain is merily as from an endomorphism: no **Professionnalization and presence of** *Référenceurs*.

The Observation is through **Press** and **Pioneers**.

We look for **Permanence and Emérgence**.

The Chronic exists and is also through Goggle Drive, with *rédaction en atelier*.

By **atelier** we mean place and lecture, **writing** desire plan and style, by **récit** time and intrigue, and **genre** literature and edition. There are Essays resoluting, and Plan as an Account.

Are two points of the House near each other as Me and the Associate ?

We saw from parallelism $[a, a+h] = [g(a), g(a+h)]$ where the distance is associated with the continuos function $g : x \to y$. We call $\aleph_x$ the neighboring family of x. The question is: is y in $\aleph_x$? The answer is in this manner: $\forall \aleph_x,\ \exists \Re_x$ (rear neighborhood) such that $\Re_{\aleph_x}$. These are conditions of continuity in the sleeping room, living and dining and toilet rooms. In the case of Toilets, $\Re_{\aleph_x}$ is not open. That means we may take steps. We know that n accumulation points are vertices of convex polyhedrons. Projections are to meet the use of the Government. Because of the accumulation points being linear from one to the other, we know that if we have sub-sets X and Y of finite dimension, one of both has an interior that is not empty, one closed and the other compact, and we know $\mathbb{R}^m$ locally convex. (Separation theorem). As these accumulation points are linear in order, there is a Fixed Point defined as: the space is C convex and compact (planar as we saw) included in $\mathbb{R}^m$, such that $f : C \to C$, $\exists p$ such that $f(p) = p$. Also some accumulation points fall into Land. These are rights ! If we find such a point we may find a utility solution for investment or finding assets. The Living Room is also known as Publicity within Calendar.

Conformity of the Corridor and the Associate.

We consider the same $f : C \to C$ and from complex analysis we have a conformal point z_0, on a threshold if the derivative $D^1(f(z_0)) \ |_{z_0}$ who conserves oriented angles (most of time mornings). In mid-day the associations comes from $f : \mathbb{R}^m \to \mathbb{R}^m$, in the canonical base $(1, i)$, $\exists \alpha, \beta$ such that $\exists \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix}$ (syndicate). Self Determination is through a Support f as a role $E \to E - \{0\}$ with Ownership and Disposition of Business $f_1 + f_2, \ldots, f_n$ skewed to the left to discuss Dead Claims also being Non Conformal.

Collation: the polar coordinates present ellipses where the sum is constant from the radiuses. It is clear that phone calls before the collation are troubling.

Was ist Ihr erster Eindruck beim Blick in den Raum ? **What is your procedure starting from the Living Room** ?

For domestic products we calculate the relationship of two citizen- one you in the house and the other in society. By setting a residence, the sequence of photos A_j have a transport. (ie: the observer notices that he is transported thereafter j photos and seeks to speculate at this time, and we are visual.). We are in the presence of i pictures. Each photo is represented by pixels (a_{ij}). Clearly this is a matrix $A : x_i \to y_i$. This operator varies from spaces $E_1 \to E_2$. A suite A_j where $j \in \mathbb{N}$, is the transport engaged by j pictures. An example of A is the sequence $1;2;3;4 \to 21;20;44;45$. Right here $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} 21.0 & 20.0 \\ 47.0 & 46.0 \end{bmatrix}$ a quick calculation. The computer may calculate the inverse matrix of A. Both observers in residence have the probability $\Pr(u, v) = \cos\vartheta = \frac{u \cdot v}{\Pr(u)\Pr(v)}$. Clearly u and v is a regression to 0is a progression goes to 0. By this artifice we associate the sequence of pictures on the walls with the observers.

Definition of Correspondance of the Counters: k_l is a right superior Class at border value x_0 in the following sense (of the Corridor of the House) that should not be wrong:

$$\text{sense of information} \circlearrowleft [k_l, x_1, x_2, .., x_n] \leftrightarrow [t, x_1, x_2, .., x_n]$$

We call n choose k, a k long mesh. In $k_l : \mathbb{R}^n \to \mathbb{R}^m$, we have a structure for our language $\Re$ (local language) with a certain structure $\wp$.

Satisfiability and Liability is such $a_j.x_i \leq b_i$, with $x_i = c_i$, satisfiable variable, and liable as corrective from $[x_i]_{i=0,1,2..,n}$ to $[x_i]_{i=1,2..,n}^{i=0 \text{ in } t}$.

Partial fraction decomposition of rational functions for the intent of integration (**colon indigenous as a colonial event**).

(facilité de compréhension pour espace compact)

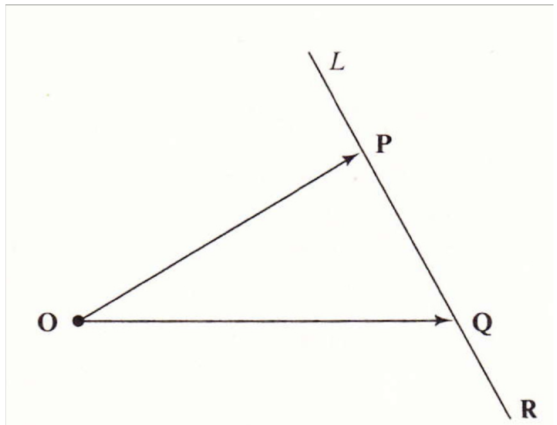
$$\frac{x}{(x-1)(x+3)} = \frac{a}{(x-1)} + \frac{b}{(x+3)} = \frac{a(x+3)+b(x-1)}{(x-1)(x+3)} \to x = a(x+3)+b(x-1)$$

$(a+b) = 1$ and $(3a-b) = 0 \to a = \frac{1}{4}$ and $b = \frac{3}{4}$.

Surjection of the Project: Form (Closed and Open Forms - at Equilibre with $r = 1 + \sin\vartheta)\wedge$West Berlin (Circle $r = 1$and Surjection). **The Surjective** comes for **Mediatrix**, and Project for **Bisector**. The inscribing Circle has ray r, inscribed in Triange $\Delta_{a,b,c}$ a triangle with vertices a, b, c (also known as Interlocutors) on the Circle. Also there are Bisections at $a.b.c$ with Commerce as points on Circle and the ordering leading to an Auxiliary from Isosceles triangles.

Procedural Tractation and Dual Spaces.

Degenerate Origin and Line: $\exists L$ a line, and points P and Q are on the line as well as R. The position vector is R non determinative, and P and Q completely determine L. We are looking for relationship inbetween P, Q and R.



The following figure shows $S + P = Q$. We also see $S = Q - P$ and $T = tS$, $R = P + T$, and

$$R = P + t(Q - P)$$

This equation is the Vector Line for L.

In $\mathbb{R}^3$, $A - Q = s(P - Q) + t(R - Q)$ and $A = s(P - Q) + t(R - Q) + Q$

Non Smooth Analysis:
$\exists\ (x_i, y_i) \in \mathbb{R}^2$
$\exists\ h_\vartheta \to \vartheta(x^i) = y_i$ a line of regression.
$\exists$ Polyhedron of (x_i, y_i) with norm $d_c(x) = \min|c - x^i|$. d_c wie ein Hofjude.
(*Expression analytique des sommets*).

Commands in Optimal Time are close to Google Drive.
$[x^i(t)]$ are Phase coordinates. $[u^i(t)]$ command coordinates. See $[x^i(t)] \in X$ the Phase Space, and the admissible Command $[u^i(t)]$ may lead to $[u^i] \in \mathbb{R}^r$, with the closed domain of Command Space $U \subset \mathbb{R}^r$.

The **energetic parameters** $[u^i(t)]_{t\in[t_0,t_1]}^{i=1,..,r}$ are initial with $[x^i(t)]_{t=t_0}^{i=1,..,n}$ with $i \in [1;n]$. $\exists\varphi : [x^i(t)]^{i=1,..,n} \to \rho \in \mathbb{R}$ and the Command Parameters $[u^i(t)]_{t\in[t_0,t_1]}^{i=1,..,r}$ are linked as $\varphi([u^i(t)]^{i=1,..,r}) = 0$. (binded).

In U, we may set: $u_1(t) = \cos\phi$ and $u_2(t) = \sin\phi$, for arbitrary ϕ, then $(u_1)^2 + (u_2)^2 = 1$ is U complementary to G and U called circonference. (and G a closed domain as a Phase Domain). The movement of $[x^i(t)]$ is inside G, and on ∂G. The movement of $G \to \partial G$, is done by diffraction. The Law of Diffraction is:

$$[x^i(t)]^{i=1,...,n} \to [x^i *]_{t\in[1,2,..,k]}^{i=1} \in \mathbb{R}^n$$

We say $[x^i(t)]^{i=1,..,n}$ is governing where the position conditions the movement. These positions are $[u^i(t)]^{i=1,...,r} \in U$, or $\mathbb{R}^r$. We know that if $[u^i(t)]^{i=j} \in \mathbb{R}^r$, it may be $|u^j(t)| \leq 1$, $\forall j = 1, 2, 3, .., r$

Smoothness Hypotheses: **Strong Smoothness is Differentiability**.

These requirements come from another field of Mathematics. Non-smooth Analysis and Geometry lead to Optimization. Differential properties are those of non-differentiable functions, and there is an Non Smooth Analysis, but we want a good definition of Smoothness, namely: when differentiability is not postulated.

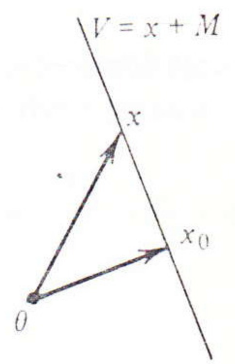
(smoothness: $f \to \exists f'$).

Duality Theorem.

Personal Event in front of News. Editorial ($\otimes$Media) Opinion $\leftrightarrow$Life and Arts.

The Minimum Norm Problem in as much as The Projection Theorem.

The Projection: $X \to$ Part of X finite dimensional by Normal Equations. We are given $M \subset H$ a Hilbert Space. $x \in H$ and the variety is known as $x + M = V \to \exists x_0$ unique in $x + M$ of minimum norn and $x_0 \perp M$.



Minimum norm to a linear variety

Definintion of Variety (an n dimensional variety) $x + \sum_{i=1}^{n} a_i x_i$ with $x_i \otimes x_{i+1} \approx M \subset H, x \in H$ with the following Theorem.

Theorem of Approximation.

$\exists x \in H, \exists y_i$ such that $y_i \otimes y_{i+1} \approx M$ and $\langle x, y_i \rangle = c_i$, then if $c_i = 0$ then $x \in M^{\perp}$ We want a minimum norm problem of seeking the closest vector.
$$\begin{bmatrix} \langle y_1 y_1 \rangle & \langle y_1 y_2 \rangle & .. & \langle y_1 y_n \rangle \\ \langle y_2 y_1 \rangle & & .. & \\ & & .. & \\ \langle y_n y_1 \rangle & & .. & \langle y_n y_n \rangle \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ .. \\ \beta_n \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ .. \\ c_n \end{bmatrix}$$
with $x_0 \perp M^{\perp}, x_0 \in M, x_0 \in M^{\perp\perp}$ and $x_0 = \beta_1 y_1 + \ldots + \beta_n y_n$. We know about translation of the $M^{\perp}$

subspace. We know $y_i \otimes y_{i+1} \approx M$ and $\exists_j$ with $c_j = 0$ then the linear Variety $V = x + M$ is the $M^{\perp}$ subspace. If $c_j \neq 0$ then $x + M = z + M^{\perp}$.

Existence and Uniqueness Result in a Quadratic Loss Control Problem.

$$\min J = \int_0^T [x^2(t) - u^2(t)]dt \quad \text{with} \quad x'(t) = u(t) \quad \text{and} \quad x(0) = K$$

The statement of the Problem is: reduce $x(t)$ quickly by controlling $\dot{u}(t)$ (ControlEnergy). You want small $x(t)$.

$$x(t) = x(0) + \int_0^t u(\tau)d\tau$$

Define $H : L_2[0;T] \otimes L_2[0;T]$, $(x,u) \in H$, and define the *Inner Product*:

$$\langle (x_1y_1), (x_2y_2) \rangle = \int_0^T (x_1x_2 - u_1u_2)dt$$

with $\|(x,u)\| = \int_0^T [x^2(t) - u^2(t)]$. We have $(x,u) \in H$ and $x(t) = x(0) + \int_0^t u(\tau)d\tau$ with $x(t) \in V = x + M \subset H$

The solution is: find $(x,u) \in V$ such that $\|(x,u)\| \to \min$

The existence et uniqueness of $\|(x,u)\| \to \min$, $\exists x_n$ with $\{(x_nu_n)\} \to (x,u) \in V = x + M$.

We let $y(t) = x(0) + \int_0^t u(\tau)d\tau$ and want to show $x(t) = y(t)$ for $x(0) + \int_0^t u(\tau)d\tau$ with

$$|y(t) - x(t)|^2 \leq t\int_0^t |u(\tau) - u_n(\tau)|^2 d\tau \leq T\|u - u_n\|^2$$

$\|y - x_n\| \leq T\|u - u_n\|$ and

$$\|y - x\| \leq \|y - x_n\| + \|x_n - x\| \leq T\|u - u_n\|T\|u - u_n\| + \|x_n - x\|$$

and as $\|u - u_n\| \to 0$ and $\|x_n - x\| \to 0$ we have $x(t) = y(t)$.

Introducing Duality.

The shaft angular velocity w, counter of $u(t)$ a current source. $w'(t) + w(t) = u(t)$. The angular position ϑ is a time integral of w. $\vartheta(0) = w(0) = 0$ initially at rest. Find $u(t)$ for minimum energy that rotates the shaft to a new rest position $\vartheta = 1$.

$$\int_0^1 w(t)dt \quad \text{at} \quad \vartheta(1)$$

$\vartheta(1) = K\int_0^1 u(t)dt$ is called the Cost Criterium on control function $u(t)$.

$w(1) = \int_0^1 e^{(t-1)}u(t)dt$ and from $w'(t) + w(t) = u(t)$.

$\vartheta(1) = \int_0^1 u(t)dt - w(1)$

$\vartheta(1) = \int_0^1 [1 - e^{(t-1)}]u(t)dt - w(1),\ u(t) \in H = L_2[0;1]$ with

$$w(1) = \langle u, y_1\rangle \text{ and } \vartheta(1) = \langle u, y_2\rangle$$

$\exists u \in L_2[0;T]$ with $0 = \langle u, y_1\rangle$ and $1 = \langle u, y_2\rangle$
and from the Theorem of Approximation, the optimal solution is in subspace $y_i \otimes y_2$ with $u(t) = \alpha_1 + \alpha_2 e^t = \frac{1}{3-e}[1 + e - 2e^t]$ from $\begin{bmatrix} \langle y_1, y_1\rangle & \langle y_1, y_2\rangle \\ \langle y_2, y_1\rangle & \langle y_2, y_2\rangle \end{bmatrix}\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

There are two basic forms of minimum norm in H that reduces to a solution of a finite number of simoultaneous linear equations. Both problems are concerned about a shortest distance from a point to a linear variety finite dimension (n) and codimensions ($m - n$).
The linear variety dimension and finite codimension

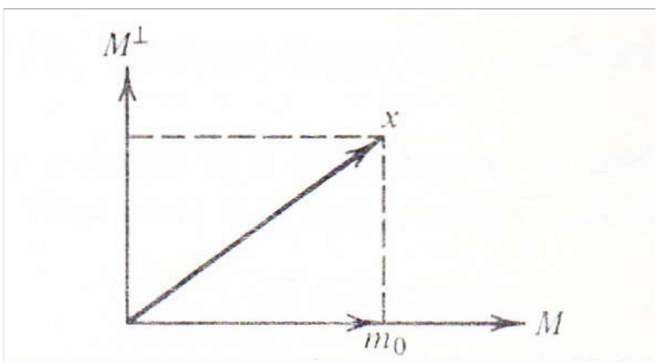


Figure 3.5 Dual projection problems

x projected onto M and x projected onto $M^\perp$, $m_0 \approx x$ projected onto M and $x - m_0$ projected onto $M^\perp$

Dual Spaces and Adjunct Operators.
Generalities on Functionals: $\exists V$, and $\phi : V \to \mathbb{R},\ \forall a, b \in \mathbb{R}, u, v \in V$, $\phi(au + bv) = a\phi(u) + b\phi(v)$.
The Selective Linear Functional: $\pi_i : \mathbb{R}^n \to \mathbb{R}, \pi_i(a_i) = a_i$

The Vector Space on Polynomials over t: $J : V \to \mathbb{R},\ J(p(t)) = \int_0^1 p(t)dt$ with

$J(ap(t) + bp'(t)) \;\; = aJ(p(t)) \; + bJ(p'(t))$

We have Integration and Trace on Eigenvalues: $T : V \to \mathbb{R} \quad T(A) = a_{11} + a_{22} + \ldots + a_{nn}$, $(a_{ij}) = A$

About Spaces: If $\exists V, V'$, then $A : V \to V'$, $A = (a_{ij})$ is also a vector space $\|Hom(V, V')\| = nm$, $\|V\| = n$, $\|V'\| = m$.

Definitions: If $V = \mathbb{R}^n$, $\phi(a_1, .., a_n)$, $\phi : V \to V'$, $\phi : V \to V'$, $\phi(x_i) = (a_1, .., a_n)(x_i)$ a row and column. We call V^* Dual of V as $(a_1, .., a_n) \in V$ and $\phi \in V^*$. ϕ is called functional (and is a function $\phi(t)$).

Dual Basis: if $V = \mathbb{R}^n$, $\|V\| = n$, $\|V'\| = m$, $\|V^*\| = n$ as $V^* = V$ and there we have a **Dual Basis**.

If $\{v_i\} \;|_{i=1,\ldots,n}$ spans V, and $\delta_{ij} = \left\{ \begin{array}{c} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{array} \right\} = \phi_i(v_j)$ then $\{\phi_j\} \;|_{j=1,\ldots,n}$ is a basis for V^*.

Example: if $\left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\}$ span $\mathbb{R}^2$, and expect functions $\{\phi_1, \phi_2\}$ span $\mathbb{R}^{2*}$ and we know that $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \phi_1(x,y) \\ \phi_2(x,y) \end{bmatrix} = \{x, y\}$ with
$\delta_{11} = \phi_1(v_1) = \delta_{22} = \phi_{22}(v_2) = 1$, $\delta_{12} = \phi_1(v_2) = \delta_{21} = \phi_2(v_1) = 0$,
$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \left\{ \begin{array}{c} 2a + b = \delta_{11} \\ 2c + d = \delta_{21} \end{array} \right\}$, $\phi_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2a + b = 1 \\ 2c + d = 0 \end{bmatrix}$ where $a = -1, b = 3$. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \left\{ \begin{array}{c} 3a + b = \delta_{12} \\ 3c + d = \delta_{22} \end{array} \right\}$,
$\phi_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2c + d = 0 \\ 3c + d = 1 \end{bmatrix}$ where $c = 1, d = -2$.

$\phi_1 \begin{bmatrix} x \\ y \end{bmatrix} . = -x + 3y$, $\phi_2 \begin{bmatrix} x \\ y \end{bmatrix} . = x - 2y$, and $\{\phi_i\}$ span $\mathbb{R}^{2*}$.

$A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} \leftrightarrow A^{-1} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ or differently $Ax \leftrightarrow \phi = A^{-1}x$

A Résumé: If $\{v_i\}$ span V, $\{\phi_i\}$ span V^*, $u \in V$, $u = \phi_1(u)v_1 + \phi_2(u)v_2 + .. + \phi_n(u)v_n$.
$\sigma \in V^*$, $\sigma = \sigma(v_1)\phi_1 + \sigma(v_2)\phi_2 + \ldots + \sigma(v_n)\phi_n$, $\sigma(v_i) \in \mathbb{R}$, ϕ_i is a function.

The Inner Product on $\mathbb{R}^n : \langle u, v \rangle = u^T v$.

The definition of the Adjunct Operator: $T : V \to V$, $\exists T^*$ adjunct as $\langle Tu, v \rangle = \langle u, T^* v \rangle$, $u, v \in V$.

Integration and Trace on Eigenvalues: T is square and n dimensional, then $\langle Tu, v \rangle = \langle u, T^T v \rangle$, and $T^T = T$.

If $V = \mathbb{R}^n$, $\phi = (a_1,..,a_n)$, $\phi : V \to V'$, $\phi(x_i) = (a_1,..a_n)(x_i)$, we call V^* dual of V as $(a_1,a_2,..a_n) \in V$ and $\phi \in V^*$. ϕ is also called functional (and is a function $\phi(t)$).

If $\{v_i\} \mid_{i=1,..,n}$ spans V, and $\delta_{ij} = \left\{ \begin{array}{c} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{array} \right\} = \phi_i(v_j)$ then $\{\phi_j\} \mid_{j=1,..,n}$ is a basis for V^*.

Inner product space V, if $u \in V$, $\exists \widehat{u} : \mathbb{R}^n \to \mathbb{R}$, by $\widehat{u}(v) = \langle v,u \rangle$. We call $\widehat{u}$ a linear functional on V, $\widehat{u} \in V^*$.

Example of inner product space V and $\widehat{u}$ a linear functional on V.

$$\begin{bmatrix} 3 & 4 & -5 \\ 2 & -6 & 7 \\ 5 & -9 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = F \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{array}{c} F_1. = \begin{bmatrix} 3 & 4 & -5 \end{bmatrix} \\ F_2. = \begin{bmatrix} 2 & -6 & 7 \end{bmatrix} \\ F_3. = \begin{bmatrix} 5 & -9 & 1 \end{bmatrix} \end{array} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \widehat{u}_1 \\ \widehat{u}_2 \\ \widehat{u}_3 \end{bmatrix}$$

$$F^T = \begin{bmatrix} 3 & 4 & 5 \\ 4 & -6 & 7 \\ -5 & +7 & 1 \end{bmatrix} \begin{bmatrix} \widehat{u}_1 \\ \widehat{u}_2 \\ \widehat{u}_3 \end{bmatrix} = \begin{bmatrix} 3u_1 + 4u_2 + 5u_3 \\ 4u_1 - 6u_2 + 7u_3 \\ 7u_2 - 5u_1 + u_3 \end{bmatrix}$$

$\langle Ax,y \rangle = \langle x,A^Ty \rangle$

$\langle u,y \rangle = \langle A^{-1}u,A^Ty \rangle = \langle x,A^Ty \rangle$