Function and non Convergent probability Frequencies and the Normal Distribution.
A selection of Photographies.

Definition of the Normal Distribution:
We say X has a normal distribution with mean $\mu \in \mathbb{R}$ and variance $\sigma^2 > 0$, a continous pdf:

$$f(x \mid \mu,\sigma^2) = \frac{\exp\left[-\frac{1}{2}(\frac{x-\mu}{\sigma})\right]}{\sqrt{2\pi}\,\sigma}$$

The z-score of the Gauβian Density or Frequency.

$$f(x) = ke^{-t^2} \to \int_0^{\frac{1}{2}} f(x)dx \approx 1$$

$$\int_\mu^i f(x)dx \Rightarrow \exists_{\mu,\sigma} \to z = \frac{i-\mu}{\sigma} \in \left[0;\quad 3.9\right] \nrightarrow Table$$

If you have a measurement $0, 3, 12 \to$ add a probability distribution.

$$\left\Vert \begin{array}{cc} x & p(x) \\ 0 & \frac{1}{3} \\ 3 & \frac{1}{3} \\ 12 & \frac{1}{3} \end{array} \right\Vert \to \mu,\sigma \text{ to be found.}$$

Example: $n = 200, \exists\bar{x} = \mu$ and σ. The question is $\Pr(X > i)$. Say $i \in \mathbb{R}$

The z-score keeps account of s.
From sample we find $s = 2000$ and $\bar{x} = 17000$ and $i = 15000$

$\bar{x} - 3s \Leftrightarrow i = 15000 \to \bar{x} = 17000 \Leftrightarrow \bar{x} + 3s$

The z-score is $\frac{15000-17000}{2000} = -1$

$$Table(1) + 0.5 = 0.841 > 0.5$$

The event may be rare, if wrong.

Advantage of the Normal Distribution: We sample form $N(0;1)$ instead of the precise unknown distributions of X then the distribution of various important explicit functional distribution parameters have a simpler form for the parameters μ and σ^2.

Most limits are in the Normal pdf and is called Convergence in Distribution, and the Central Limit Theorem says: $X_i \mid_{i=1,2,..,n} \to \exists\mu,\sigma^2 < \infty$ then $\forall x$ fixed:

$\lim_{n\to\infty} \Pr\left[\frac{\sqrt{n}\,(\overline{X}_n - \mu)}{\sigma} \leq x \right] \equiv N(0,1)$. (function of non normal *pdf* will have approximately normal distribution). To justify why μ and σ are well put, we study the moment generation function: The k moments are defined as $E(X^k)$, where the first moment is $E(X) = \mu$ and $E(|X|^2) < \infty$ the Variance σ^2. Why ? We define the moment generation function $\psi(t) = E(e^{tX})$, $\psi'(0) = \left[\frac{\partial}{\partial t} E(e^{tX}) \right]_{t=0} = XE(e^{tX})_{t=0} = E(X)$. Also $E(X^2) = \psi''(0) = 2$, is with $\psi^{(n)}(0) = E(X^{(n)})$. For the $N(0;1)$, we have $\psi'(t) = \frac{1}{(1-t)^2}$ and $\psi''(t) = \frac{2}{(1-t)^3}$ to $E(X) = \psi'(0) = 1$, and $V(X) = \psi''(0) - [\psi'(0)]^2$.

Calculating the Moment Generating Function:
From *pdf* we have→ $f(x) = e^{-x} \mid_{x>0}$ and $f(x) = 0 \mid_{x\leq 0}$.
$\forall t, \exists \psi(t) = E(e^{tX}) = I = \int_0^\infty e^{tx}e^{-x}dx = \int_0^\infty e^{(t-1)x}dx < \infty$ iff $t < 1$. We know $\psi(t) = \frac{1}{1-t}$, where $\psi(t) \mid_{t=0} < \infty \Rightarrow \exists \psi'(t), \exists \psi''(t)$. The Uniqueness of the Moment Generation Function is stated as $\exists mgf$ of X_1 and X_2 $\forall t$ close to 0, and then the *pdf* of X_1 and X_2 are identical.

The Central Limit Theorem.
$X_i \mid_{i=1,2,..,n} \to \exists \mu, \sigma^2 < \infty$ then $\forall x$ fixed: $\lim_{n\to\infty} \Pr\left[\frac{\sqrt{n}\,(\overline{X}_n - \mu)}{\sigma} \leq x \right] \equiv N(0,1)$.
(function of non normal *pdf* will have approximately normal distribution). (convergence in distribution). This theorem addresses $X_i \mid_{i=1,2,..,n} \to \exists \mu, \sigma^2 < \infty$ then $\forall x$ fixed in $\lim_{n\to\infty} \Pr\left[\frac{\sqrt{n}\,(\overline{X}_n - \mu)}{\sigma} \leq x \right] \equiv N(0,1)$. It may be extended to Sum of independent random variables even though that it is a Sum of non Normal distributions, we say differing form $N(0;1)$. The Sum is $\sum_{i=1}^{n} X_i$.